K22P 0193

Reg. No. : ....

Name : ....

II Semester M.Sc. Degree (CBSS – Reg./Supple./Imp.) Examination, April 2022
(2018 Admission Onwards)
MATHEMATICS
MAT 2C10 : Partial Differential Equations and Integral Equations

Time : 3 Hours   Max. Marks : 80

PART – A

Answer **any four** questions from this Part. **Each** question carries **4** marks.

1. Eliminate the arbitrary function F from $z = F\left(\frac{xy}{z}\right)$ and find the corresponding partial differential equation.

2. Find the general solution of yzp + xzq = xy.

3. Show that the solution of the Dirichlet problem if it exists is unique.

4. Find the Riemann function of the equation $Lu = u_{xy} + \frac{1}{4}u = 0$.

5. Transform the problem y″ + xy = 1, y(0) = 0, y(l) = 1 into an integral equation.

6. Prove that the characteristic numbers of a Fredholm equation with a real symmetric kernel are all real. **(4×4=16)**

PART – B

Answer **four** questions from this Part, without omitting **any** Unit. **Each** question carries **16** marks.

Unit – 1

7. a) Show that the Pfaffian differential equation $\vec{X}.\vec{dr} = P(x, y, z)dx + Q(x, y, z)dy + R(x, y, z)dz = 0$ is integrable if and only if $\vec{X}.\text{curl}\,\vec{X} = 0$.

   b) Show that ydx + xdy + 2zdz = 0 is integrable and find its integral.

8. a) Find a complete integral of $z^2 - pqxy = 0$ by Charpits method.

   b) Solve $u_x^2 + u_y^2 + u_z = 1$ by Jacobi's method.

9. a) Find a complete integral of the equation $(p^2 + q^2)\, x = pz$ and the integral surface containing the curve $C : x_0 = 0,\ y_0 = s^2,\ z_0 = 2s$.

   b) Solve $xz_y - yz_x = z$ with the initial condition $z(x, 0) = f(x),\ x \geq 0$.

Unit – 2

10. a) Reduce the equation $u_{xx} + 2u_{xy} + 17u_{yy} = 0$ into canonical form.

    b) Derive d'Alembert's solution of one dimensional wave equation.

11. a) Solve $y_{tt} - C^2 y_{xx} = 0,\ 0 < x < 1,\ t > 0$.

    $y(0, t) = y(1, t) = 0$

    $y(x, 0) = x(1 - x),\ 0 \leq x \leq 1$

    $y_t(x, 0) = 0,\ 0 \leq x \leq 1$

    b) State and prove Harnack's theorem.

12. a) Solve the differential equation corresponding to heat conduction in a finite rod.

    b) Prove that the solution $u(x, t)$ of the differential equation

    $u_t - ku_{xx} = F(x, t),\ 0 < x < l,\ t > 0$ satisfying the initial condition

    $u(x, 0) = f(x),\ 0 \leq x \leq l$ and the boundary conditions

    $u(0, t) = u(l, t) = 0,\ t \geq 0$ is unique.

Unit – 3

13. a) Solve $y'' = f(x),\ y(0) = y(l) = 0$.

    b) Solve $x^2 \dfrac{d^2y}{dx^2} + x\dfrac{dy}{dx} + (\lambda x^2 - 1)y = 0$ with $y(0) = 0,\ y(1) = 0$.

14. a) If y_m, y_n are characteristic functions corresponding to different characteristic numbers λ_m, λ_n of $y(x) = \lambda \int_0^1 K(x,\xi) y(\xi)\, d\xi$, then if K (x, ξ) is symmetric. Prove that y_m and y_n are orthogonal over (a, b).

b) Solve the integral equation $y(x) = f(x) + \lambda \int_0^1 (1 - 3x\xi) y(\xi)\, d\xi$ and discuss all its possible cases.

15. a) Describe the iterative method for solving Fredholm equation of second kind.

b) Find the iterated Kernels $K_2(x, \xi)$ and $K_3(x, \xi)$ associated with $K(x, \xi) = |x - \xi|$ in the interval [0, 1]. **(4×16=64)**